EFFECTS OF QUADRATURE SIGNALS ON SERVO SYSTEMS

The usual arrangement of the Digital to Synchro converter in a control loop is shown in Chapter 4, Fig. 4-38. The signal from the control transformer is amplified and fed into a Phase sensitive detector (PSD). The DC output from the PSD forms the error signal for the control loop.

As can be seen from Fig. 4-38, the PSD is driven from the reference signal and the operation of the PSD is such as to give a gain of + 1 when the reference is positive and a gain of -1 when the reference signal is negative. Simple consideration of the PSD will show that, if an AC voltage at the same frequency as the reference voltage but shifted in phase by 90 degrees is introduced in series with the usual input from the CT, no errors will be caused. (Errors could however be caused by the injection of such an AC voltage, if the voltage was of sufficient magnitude to cause asymmetric limiting in the amplifiers). Such an injection of a 90^0 phase shifted signal *is not the same* as the introduction of a real quadrature voltage caused by different phase shifts in the signal paths.

The following simple analysis shows the errors due to quadrature and how these errors are increased by a signal to reference phase shift. The signals will be considered in Resolver form to reduce the number of terms in the equations.

Referring to Fig. F-1, we will consider the reference to be in phase with the input signals and then introduce a small phase shift on one signal relative to the other.

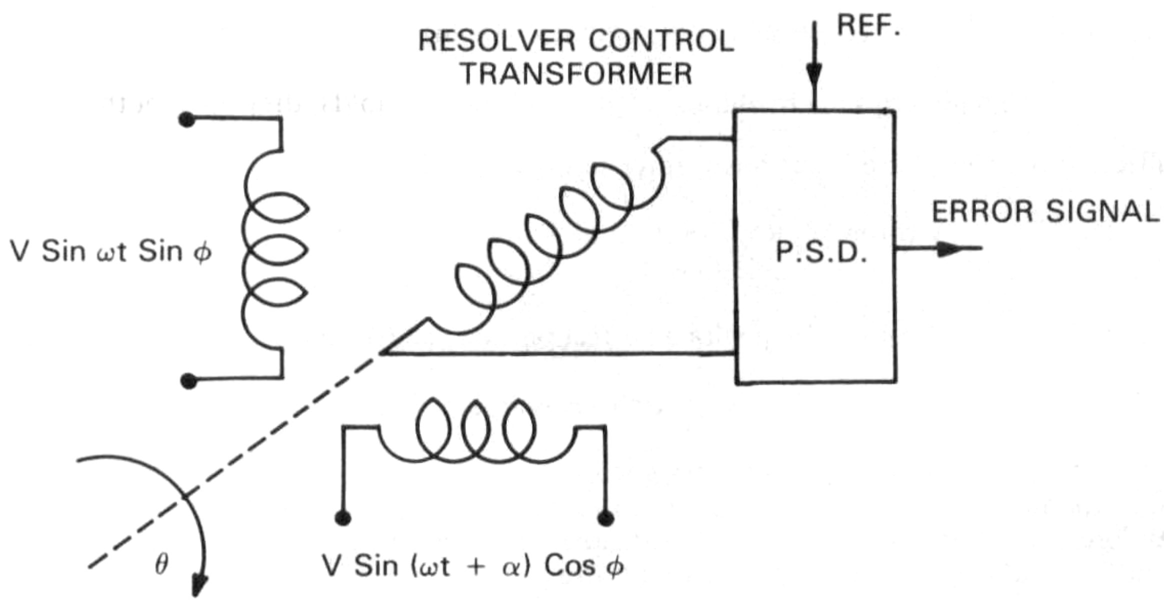

Fig. F-1 A resolver with a differential signal phase shift a.

In a perfect system the voltages would be,

$$Vs = V \sin \omega t . Sin\phi$$
$$Vc = V \sin \omega t . Cos\phi$$
$$V = V \sin \omega t$$

To show how the quadrature terms arises a phase shift of a will be applied to the carrier in Ye. The three voltages are now:

$$Vs = V \sin \omega t . Sin\phi$$
$$Vc = V \sin(\omega t + \alpha) . Cos\phi$$
$$V = V \sin \omega t$$

At a fixed angle θ the Resolver behaves like two transformers with a common secondary where the ratios are:

R Cos θ → Sine input to rotor
and - *R Sin θ* → Cosine input to rotor

The equivalent R for the Synchro control transformer type 11CT4L, (9Ov L-L) is 0.64.
The voltage in the rotor winding will be the sum of the two voltages i.e.

$$Vrotor = (VSin\omega t.sin\phi)\ R\ Cos\ \theta - [Vsin\ (\omega t + \alpha)\ Cos\ \phi]\ R\ Sin\ \theta \qquad (1)$$

Where ϕ is the digital angle and θ is the angle of the resolver shaft and a is the phase shift introduced into the Cosine channel.

In equation (1) the next step is to expand the term

$$Sin\ (\omega t + \alpha)$$

by using the relationship:

$$Sin\ (A + B) = Sin\ A\ Cos\ B + Cos\ A\ Sin\ B$$

Doing this we get,

$$V\ rotor = RV[Sin\ \omega t^{*}.\ Sin\ \phi.\ Cos\ \theta - Cos\phi.\ Sin\theta\ (Sin\omega t.Cos\ \alpha + Cos\omega t\ {}^{**}Sin\alpha)]$$

*These terms are in phase. ** This is the quadrature term.

Collecting the "in-phase" and quadrature terms we have:

$$V\ rotor = RV[Sin\omega t\ (Sin\phi.Cos\ \theta - Cos\phi.Sin\theta.Cos\alpha) \qquad \ldots(2)$$
$$+ Cos\omega t(Cos\phi.Sin\theta.Sin\alpha)]$$

A properly designed control loop with a perfect phase sensitive detector will ignore the term:

$$Cos\omega t(Cos\phi.Sin\theta.Sin\alpha)$$

in equation (2). For such a properly designed servo loop there is an error which is very small due to the fact that Cos α in the first term of equation (2) is not quite equal to 1.
Before proceeding to the case of the quadrature term itself we will find what the error would be in a loop which rejects the quadrature.
We make use of the fact that

$$Cos\alpha = 1 + \frac{\alpha^2}{2} + \frac{\alpha^4}{4!} \ldots\ldots\ldots\ldots. \quad (\alpha \text{ in radians})$$

and for small α (which we are considering)

$$Cos\alpha = 1 - \frac{\alpha^2}{2}$$ where α is in radians.

Since we are concerned for the present with a good loop which rejects the quadrature signal, we can ignore that term in the error signal. Doing this and substituting for *Cos* α as above gives:

$$Vrotor = RV \{Sin\omega t.[Sin\phi.Cos\theta - (1 - \alpha^2/2)\ Sin\ \theta.Cos\ \phi]\}$$

The servo will alter θ to make *Vrotor = zero*.

i.e. $$0 = [Sin\phi.Cos\theta - Sin\theta.Cos\phi + \alpha^2/2\ Sin\ \theta.Cos\ \phi] \quad (3)$$

In equation (3), it is the term

$$\frac{\alpha^2}{2}\ Sin\ \theta.\ Cos\ \phi$$

which gives rise to the error.

For $\phi \approx \theta$, $Sin\theta.\ Cos\ \phi$ has a maximum of 0.5 at $\theta \approx 45^0$

And since $Sin\ \phi\ Cos\ \theta - Sin\ \theta\ Cos\ \phi \cong (\phi - \theta)$ radians, the error in radians is:

$$\varepsilon = \frac{\alpha^2}{2} \times 0.5$$ (α and error are in radians)

$$\text{error in degrees} = \frac{\alpha^2 \times 0.5}{57 \times 2}$$ (α in degrees).

$$\text{error in minutes} = \frac{60 \times 0.5\alpha^2}{57 \times 2}$$ (α in degrees

$$\approx \frac{\alpha^2}{4}$$ (α in degrees).

i.e. ¼ arc minute for 10 of phase at worst angle of 45^0

We now look at the quadrature signal which should be rejected. It is the term

$RV\ Cos\ \omega\ t\ [Cos\ \phi\ sin\ \theta\ Sin\ \alpha]$ of equation (2)

For $\phi \approx \theta$, the maximum $Cos\ \phi\ Sin\ \theta$ is for $\phi \approx \theta = 45^0$ and the value is 0.5.

The peak value of the quadrature is

Quadrature = α RV x 0.5 (α is in radians) (Sin $\alpha \approx \alpha$)
V = $90\sqrt{2}$
R = 0.64 for the 11CT4b Control Transformer (90 volts L-L)
and for α = 1/57 (1 degree).

The rms value of the quadrature on the rotor is $\frac{90 \times 0.64 \times 0.5}{57}$ volts per degree.

rms Quadrature Voltage on rotor = 0.5 volts per 1 degree of differential phase shift.

i.e. For l00mV rms the differential phase has to be 0.2 degrees.

The foregoing example of the effect of quadrature was based on the assumption that the signals and reference were in phase before the small differential phase shift was introduced. In practical systems there is often a phase shift of several degrees between the signals and reference. This phase shift should be minimised in the design of Synchro systems. Both control transmitter and control receiver Synchros give rise to phase leads between the input and output voltages and balancing phase leads are often introduced into the reference before it is used to drive the phase sensitive detector. DSC's generally give negligible phase shift between the input reference and output signals, if therefore a CX is replace by a DSC, the phase angle between the CT output voltage and the reference will be changed and a corresponding correction should be put in to the system at some point.

The following simple analysis shows the effects of quadrature signals in the presence of signal to reference phase shift. Again to reduce the number of terms in the equations the resolver example will be considered.

The effect of reference to signal phase shift with quadrature

Let the resolver form signals be:

$$Sin\ \omega t\ Cos\ \phi \text{ and } Sin\ (\omega t + \alpha)\ Sin\ \phi$$

The control resolver rotor voltage will be:

$$Sin\omega t.\ Cos\ \phi.Sin\theta - Sin\ (\omega t + \alpha).Sin\ \phi.\ Cos\ \theta$$

where θ is the shaft angle.

The action of the phase sensitive detector with a phase shifted reference where the phase shift is β is given by integrating the resolver rotor output with respect to ωt from $\omega t = \beta$ to $\omega t = \beta + \pi$. The control loop will set the result to zero i.e.:

$$Cos\phi.Sin\theta\int_{\beta}^{\beta+\pi} Sin\omega t.d\omega t - Sin\phi.Cos\theta\int_{\beta}^{\beta+\pi} Sin\omega t.d\omega t = 0$$

Expanding *Sin* $(\omega t + \alpha)$ and integrating gives:

$$Cos2.Sin3.[Coswt].^{\omega t=\beta+\pi}_{\omega t=\beta} + Sin\phi.Cos\theta.Cos\alpha.[Cos\omega t]^{\omega t=\beta+\alpha}_{\omega t=\beta} - Sin\phi.Cos\theta.[Sin\omega t]^{\omega t=\beta+\pi}_{\omega t=\beta} = 0$$

or

$$-2.Cos\phi.\ Sin\theta\ Cos\beta + 2Sin\phi\ Cos\theta.Cos\alpha.Cos\beta - 2Sin\phi.Cos\theta.Sin\alpha\ Sin\beta = 0$$

Now both β and α are small so:

$$Cos\alpha \approx 1 - \frac{\alpha^2}{2} \text{ and } Sin\ \beta \approx \beta \text{ may be used.}$$

where α and β are in radians.

Making these substitutions and rearranging the terms gives:

$$\left(1 - \frac{\beta^2}{2}\right).Sin(\phi - \theta) - \left(1 - \frac{\beta^2}{2}\right)\frac{\alpha^2}{2}.Sin\phi.Cos\theta - \alpha\beta.Sin\phi.Cos\theta = 0$$

The second and third terms in this expression are the ones which cause the error. The coefficients Sinϕ , Cos θ which occur in both these terms have a maximum value of 0.5 for $\phi \approx \theta$. Substituting 0.5 for Sinϕ , Cos θ to give the maximum error (which is for $\theta \approx \phi$ = 45 degrees) gives:

$$\left(1-\frac{\beta^2}{2}\right).Sin(\phi-\theta)-0.5[\left(1-\frac{\beta^2}{2}\right)\frac{\alpha^2}{2}.+\alpha\beta]=0$$

And this formula can be used to calculate the error, as an example for

$$\alpha=\frac{1}{57} \text{ and } \beta=\frac{5}{57}$$

i.e. 1 degree differential phase shift (which is large) and +5 degrees phase shift on the reference.

The error is very nearly equal to:

0.5 $\alpha\beta$ radians

or $\frac{0.5\times 5}{57\times 57}$ radians, for 1^0 differential and 5^0 reference phase shift.

or $\frac{0.5\times 5}{57}$ degrees

or $\frac{0.5\times 5\times 60}{57}$ arc minutes

or $\approx 0.5\times 5=2.5$ arc minutes

This figure compares with the earlier figure of ¼ arc minute for the case with no reference phase shift.

It should be understood that the 1^0 differential phase shift taken in these examples is very large. Quadrature voltages in converters and CTs jointly is more likely to correspond to an equivalent phase shift of 0.2 degrees or less, i.e. the quadrature due to both the DRC and resolver will be less than 100 millivolts.